Binary Search Algorithm

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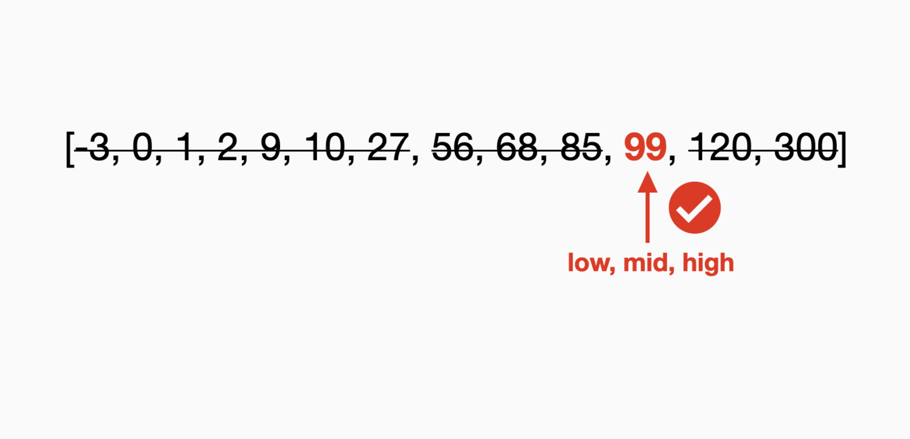
Overview

1. Understand binary search (example)
2. Design the algorithm (pseudocode)
3. Algorithm analysis (time complexity)
4. Binary search's applications
5. Binary search's limitations and alternatives

What is binary search?

A **super** efficient algorithm for finding target value within a **sorted** array.

Binary search example



[~~-3, 0, 1, 2, 9, 10, 27~~, ~~56, 68, 85~~, 99, ~~120, 300~~]

Design the binary search algorithm (pseudocode)

```
Start searching the entire array using lowest and highest indices.
Repeat the following steps until the search range is empty:
    Find the middle position of the current search range.
    If the middle value is less than the target value:
        Narrow the search range to the higher half.
    Else if the middle value is greater than the target value:
        Narrow the search range to the lower half.
    Else (the middle value equals the target):
        Find the target and return its index.
The search range becomes empty, return -1 indicating the target is not found.
```

Note: Algorithm implementations posted on the course GitHub.

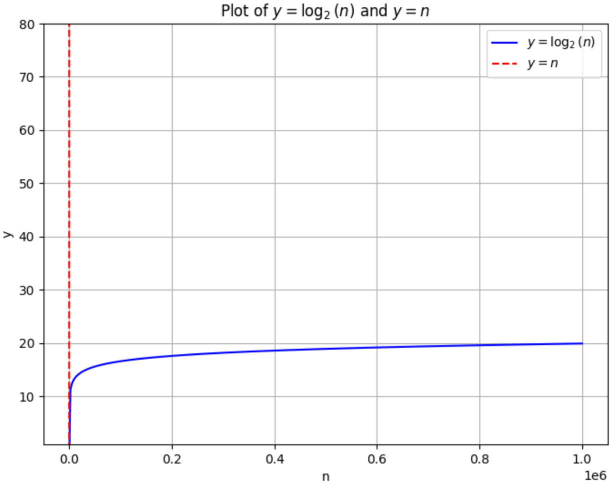
Algorithm analysis (time complexity - worst case)

$$\begin{aligned}
T(n) &= T(n/2) + 1 \\
&= T(n/4) + 1 + 1 \\
&= T(n/8) + 1 + 1 + 1 \\
&\vdots \\
&= T(2 \text{ or } 3) + 1 + \cdots + 1 \\
&= T(1) + 1 + 1 + \cdots + 1 \\
&= 1 + \log_2 n \\
&= \mathcal{O}(\log_2 n)
\end{aligned}$$

Feel the time complexity - King’s Chessboard

1	2	4	8	16	32	64	128
256	512	1024	2048	4096	8192	16384	32768
66k	131k	262k	524k	1.0m	2.1m	4.2m	8.4m
16.8m	33.6m	67.1m	134m	268m	537m	1.1b	2.1b
1.1t							
							9.2×10^{18}

Feel the time complexity



Binary search's applications

- **Guess game**
- **Search in a dictionary**
- **Find a nearest target** (LeetCode practices)
- **Root search of a function**
- **git bisect**
- **Interpolation search** (data roughly uniformly distributed)
- **Exponential search** (data unbounded)
- **Data structure: Binary Search Tree, B Tree**

Binary search's limitations and alternatives

Limitations:

sorted and **array**.

Alternatives:

1. **Linear search**
2. **Interpolation search**
3. **Exponential search**
4. **Hash table**: when order is not required.
5. **Binary search tree (self-balanced)**: AVL tree, red black tree.

Q & A

References

- Website:
  ”https://learn.zybooks.com/zybook/CAPILANOUCOMP120Fall2021”

- Thomas H. Cormen, et al. (2009)
  Introduction to Algorithms
  3rd Edition, MIT Press